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The string scale and the Planck scale

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Abstract

A particle spectrum below the string scale in accordance with predictions from heterotic string theory yields a Planck mass $m_{Pl} = (8\pi G_N)^{-1/2}$ which exceeds the string scale by a factor $\simeq 61.9$. A Planck mass $m_{Pl} = 2.43 \times 10^{18}$ GeV then corresponds to a string scale $m_s = 3.9 \times 10^{16}$ GeV. Such a low value for the string scale in turn implies that the relative strength of graviton and vector exchange in the string/M-theory phase exceeds the corresponding ratio in the low energy field theory.

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Among all contemporary attempts to extend our knowledge all the way up to scales where quantum gravitational effects are expected to play a role string theory provides the most advanced and best developed theoretical framework. In recent years the field was turmoiled by the emergence of M-theory as a non-perturbative formulation eventually unifying all known consistent string theories [1] (see also [2] for early work on the role of eleven dimensions in string theory and the appearance of matrix models in membrane dynamics), and one may ask whether string theory can be expected to provide any answers to string scale questions in its present form. However, the subject develops at a fast pace from a phase of existence proofs into a phase where dynamical issues can be addressed [1,3,4] although some conjectural dualities between different sectors of the theory still await further confirmation. For these reasons it might be more appropriate to denote the transition scale between energies where low energy field theory formulations apply and M-theory as an M-scale, but here I will still use the notion of string scale. I should also point out that in the present letter I'm only considering the string scale from the low energy perspective, asking at which temperature we might expect resolution of extended objects. Therefore all assertions in the present paper only rely on the assumption that all low energy field theories arising from M-theory are supergravity and those supergravity-Yang-Mills-systems inherited from the known consistent superstring theories. If another field theory limit would emerge with a larger low mass field content than the heterotic string this would increase the ratio m_{Pl}/m_s above the value calculated here.

Since we will consider the string scale from a low energy perspective no genuine M-theoretic tools will be needed in the present paper and I would like to refer the reader to some very useful recent reviews on duality and M-theory for the high energy formulation of the theory [5]. From a phenomenological point of view it still seems reasonable to assume that the transition to low energy field theories proceeds through the heterotic string spectrum [6–8], and here I would like to point out that this gives an interesting new estimate on the relation between the string scale and the Planck scale from a thermodynamical point of view. It also turns out that simple thermodynamical properties of the transition from M-theory to a field theory description favor the heterotic phase.

The connection between the string scale, the Planck scale and the GUT scale is a long standing issue in string theory. The role of a separate GUT scale always remained somewhat obscure and was rather puzzling from a string theory perspective. In the present letter I would like to point out that the large number of degrees of freedom of the heterotic string implies a remarkable coincidence of the string scale with the usual estimate on the GUT scale from supersymmetric β functions [9]. However, I would also like to stress that Witten recently proposed another intriguing solution to the GUT scale puzzle: In [3] Witten points out that identification of the GUT scale with a Kaluza–Klein scale describing compactification from eleven to five dimensions and subsequent compactification from five to four dimensions at a scale somewhat below m_{GUT} yields a ratio for m_{Pl}/m_{GUT} in agreement with the data if the eleven-dimensional Planck mass $m_{Pl,11}$ is slightly above the GUT scale ($m_{Pl,11} \simeq 2m_{GUT} \simeq 14m_{5\to 4}$ would work, e.g.). In this scenario the GUT scale looks like a Kaluza–Klein scale rather than a string scale, but this is only an artifact of the long wavelength approximation. In the present approach, on the other hand, field theoretical compactifications play no role, and it will be very interesting to further elucidate the relationship between the field theoretic investigations in [3] and the present considerations based

on heterotic string thermodynamics.

Heterotic string theory made its first contact with M-theory through the identification with M-theory compactified on an interval [10]. This observation established a link to string inspired phenomenology and boosted conjectures that M-theory may really provide a unifying framework for all consistent superstring theories. Among all known field theory formulations of string theory or M-theory the heterotic $E_8 \times E_8$ theory is unique for its interesting gauge sector and its large number of massless degrees of freedom. In the particle sector the heterotic string predicts 4032 bosonic massless degrees of freedom and the same number of fermionic degrees of freedom, and the effective number of relativistic degrees of freedom amounts to $g_*(T_s) = 7560$. This exceeds the effective number of relativistic type *II* or type *I* degrees of freedom $g_{*II}(T_s) = 2g_{*I}(T_s) = 240$ considerably, and as a consequence the heterotic string predicts the lowest value of the string scale for a given value of the Planck scale:

The early phase of the universe below the string scale is radiation dominated and evolving back present energy densities we know that curvature contributions are negligible during radiation dominance. The energy density during this early phase is then

$$\varrho = \frac{3m_{Pl}^2}{4t^2} = \frac{\pi^2}{30} g_*(T) T^4 \tag{1}$$

where $m_{Pl} = (8\pi G_N)^{-1/2}$ is the reduced Planck mass and t is the time parameter in the Friedmann–Robertson–Walker line element.

Eq. (1) tells us how the string scale $m_s = 1/t_s$ relates to the string temperature and m_{Pl}:

$$m_{Pl} m_s = 57.6\, T_s^2 \tag{2}$$

and these simple thermodynamical properties of the heterotic string provide a strong hint for coincidence of the GUT scale with the string scale: We expect that the extension of strings becomes visible when the energy per degree of freedom is powerful enough to resolve the string length t_s. In the relativistic domain the average energy ϵ per physical degree of freedom relates to the temperature via $T \simeq 1.037\epsilon$, and inserting this and the value of the reduced Planck mass $m_{Pl} = 2.43 \times 10^{18}$ GeV in (2) gives

$$m_s = \frac{m_{Pl}}{61.9} = 3.9 \times 10^{16}\,\text{GeV}. \tag{3}$$

This simple calculation also shows that the string scale for type *II* theories and type *I* theory or compactified 11D supergravity would have to be higher to explain the same measured value of the Planck scale. However, from the string theory point of view the scale m_s is more fundamental than m_{Pl}, the latter being fixed through the string scale and the requirement that the transition to the field theory description has to emerge self-consistently at an energy density where string lengths can be resolved (2). Therefore, if we would not see low energy remnants of a particle spectrum inherited from heterotic string theory but rather detect remnants of type *II*, type *I*, or SUGRA spectra, we would see a smaller value of the Planck mass: Gravitational systems are stronger bound in these phases.

The matching condition may also explain the dominance of the heterotic phase in the low energy regime: Assuming that the energy density of the universe decreases from a high value above the heterotic or type *II* string scale, which field theory should we expect to

emerge? To me it seems reasonable to assume that field theory to take over which satisfies the condition

$$\varrho = \frac{\pi^2}{30} g_*(T_s) T_s^4 \tag{4}$$

for transition to a field theory description first. However, the theory which satisfies this condition for the highest possible energy density for given string length is the theory which contains most relativistic degrees of freedom, i.e. the heterotic string.

In deriving the ratio (3) between the reduced Planck mass and the string scale I assumed that the field theory emerging at the string scale is four-dimensional from the outset. If there is an intermediate ten-dimensional field theory phase below the string scale and above a separate Kaluza–Klein scale m_{KK} this scale will also show up in the matching condition at the string scale. The dependence of m_{Pl}/m_s on $(m_s/m_{KK})^{(n-3)/2}$ is in fact the strongest impact of an intermediate $(n+1)$-dimensional field theory phase on the string scale. To elucidate this consider radiation dominated Friedmann cosmology in $D = n+1$ dimensions: The energy density ϱ and the scale factor $\mathcal{R}$ follow from

$$\dot{\mathcal{R}}^2 = \frac{2\kappa_D}{n(n-1)} \mathcal{R}^2 \varrho,$$

$$\frac{d\varrho}{\varrho} = -(n+1)\frac{d\mathcal{R}}{\mathcal{R}},$$

where κ_D multiplies the energy momentum tensor in the D-dimensional Einstein equations. These equations are readily integrated to yield

$$\varrho = \frac{2n(n-1)}{(n+1)^2 \kappa_D t^2}. \tag{5}$$

The thermodynamic expressions follow from the corresponding phase space integrals in n spatial dimensions

$$\varrho = g_* \frac{n!\zeta(n+1)}{2^{n-1}\sqrt{\pi}^n \Gamma(n/2)} T^{n+1}, \tag{6}$$

where bosonic and fermionic degrees of freedom contribute according to

$$g_* = g_B + \left(1 - \frac{1}{2^n}\right) g_F.$$

The difference between the average energy per relativistic degree of freedom and spatial direction ϵ and the temperature

$$\epsilon = \frac{2^{n+1}-1}{2^{n+1}-2} \frac{\zeta(n+1)}{\zeta(n)} T$$

does not have a strong impact, but is nevertheless taken into account.

The inverse compactification volume rescales $1/\kappa_D$ to the reduced Planck mass via $1/\kappa_D = m_{Pl}^2 m_{KK}^{n-3}$, and we can not directly infer the ratio between m_s and m_{Pl} without

further information on m_{KK}. In ten dimensions the effective number of degrees of freedom of the heterotic string is $g_{*10} = 8056.125$ and the resulting estimates on the string scale with an intermediate ten-dimensional phase is

$$m_{s10} = 3.9 \times 10^{16}\,\mathrm{GeV} \times (m_{KK}/m_{s10})^3.$$

However, M-theory in its current stage of evolution indicates that there is nothing special about 1-branes in ten dimensions, and on a field theoretic level neither string theory nor M-theory require an intermediate ten-dimensional field theory phase to occur. For this reason and because compactification is really hard to achieve in a purely field theoretic setting I consider a direct transition to a four-dimensional field theory as the most interesting option, thus favoring a string scale (3). After all M-theory will have to tell us the number of dimensions to occur in the transition to field theory.

How can it be that heterotic string thermodynamics gives such a low value for the string scale which was considered impossible before? Gross *et al.* and Ginsparg had already pointed out that the string and the gauge coupling should relate to the ratio between the string scale and the Planck scale according to $m_s \simeq g m_{Pl}$ [7,11], and Kaplunovsky had addressed the corresponding threshold corrections to string unification [12].

Inserting our result into this relation would give too low a value for g. However, the indentification of g with the ratio m_s/m_{Pl} was derived under the assumption that low energy gravitational and gauge interactions can alternatively be described in terms of string graviton and vector exchange, thus matching the string coupling to the Planck scale. In the present paper, on the other hand, the Planck mass in the first place appears as a constant of proportionality between the energy density ϱ_c needed to resolve a string and the string length squared. From the low energy point of view ϱ_c is much higher than m_s^4 because there are so many degrees of freedom in the heterotic phase to absorb energy before the heat bath can resolve strings or other extended objects. From the high energy point of view ϱ_c is much higher than m_s^4 because there are so many degrees of freedom in the heterotic phase to transfer energy into field theory degrees of freedom. Reinspection of the calculations of Gross *et al.* and Ginsparg in the light of Eq. (3) implies that in the M-theory phase graviton exchange is stronger in comparison to vector exchange than in the field theory phase by a factor m_{Pl}/m_s, and this may shed new light on the relation between M-theory interactions and low energy gauge and gravitational interactions. This also indicates that the string scale is not just a scale where a field theory approximation to M-theory becomes poor but rather corresponds to a phase transition.

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